NASA TECHNICAL TRANSLATION

NASA TT F-485

c.1

NASA TT F-485

VISCOUS HYPERSONIC FLOW PAST A SLENDER CONE

by V. S. Nikolayev

Inzhenernyy Zhurnal,
Volume II, No. 3, 1962

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MAY 1967

VISCOUS HYPERSONIC FLOW PAST A SLENDER CONE

By V. S. Nikolayev

Translation of "Obtekaniye tonkogo konusa vyazkim giperzvukovym potokom."
Inzhenernyy Zhurnal, Vol. II,
No. 3, pp. 9-13, 1962.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

CUT ALONG THIS LINE

FOLD LINE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546

OFFICIAL BUSINESS

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS & SPACE ADMINISTRATION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
CODE USS-T
WASHINGTON, D.C. 20546

NASA TTF No.
485

FOLD LINE

CUT ALONG THIS LINE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL TRANSLATION EVALUATION

Budget Bureau No. 104-R037
Approval Expires: Sept. 30, 1969

TO:
THE USERS OF THIS TRANSLATION →

NASA TTF NO. 485

MAINTAINING THE QUALITY OF NASA TRANSLATIONS REQUIRES A CONTINUOUS EVALUATION PROGRAM. PLEASE COMPLETE AND MAIL THIS FORM TO AID IN THE EVALUATION OF THE USEFULNESS AND QUALITY OF THE TRANSLATING SERVICE.

THIS PUBLICATION *(Check one or more)*

- [ ] FURNISHED VALUABLE NEW DATA OR A NEW APPROACH TO RESEARCH.
- [ ] VERIFIED INFORMATION AVAILABLE FROM OTHER SOURCES.
- [ ] FURNISHED INTERESTING BACKGROUND INFORMATION.
- [ ] OTHER *(Explain)*: ____________

FOLD LINE **FOLD LINE**

TRANSLATION TEXT *(Check one)*

- [ ] IS TECHNICALLY ACCURATE.
- [ ] IS SUFFICIENTLY ACCURATE FOR OUR PURPOSE.
- [ ] IS SATISFACTORY, BUT CONTAINS MINOR ERRORS.
- [ ] IS UNSATISFACTORY BECAUSE OF *(Check one or more)*:
  - [ ] POOR TERMINOLOGY.
  - [ ] NUMERICAL INACCURACIES.
  - [ ] INCOMPLETE TRANSLATION.
  - [ ] ILLEGIBLE SYMBOLS, TABULATIONS, OR CURVES.
  - [ ] OTHER *(Explain)*: ____________

FOLD LINE **FOLD LINE**

REMARKS

FROM

DATE

NOTE: REMOVE THIS SHEET FROM THE PUBLICATION, FOLD AS INDICATED, STAPLE OR TAPE, AND MAIL. NO POSTAGE NECESSARY.

NASA Form 1332 AUG 66

GPO 915-777

CUT ALONG THIS LINE

CUT ALONG THIS LINE

VISCOUS HYPERSONIC FLOW PAST A SLENDER CONE /9*

V.S.Nikolayev

The hypersonic flow of a perfect gas at zero angle of attack past a slender cone is calculated, allowing for the transverse curvature of the surface and the interaction of the boundary layer with the shock wave. The integral relations of impulse and energy are utilized in calculating the boundary layer, assuming the profile of the longitudinal component of velocity as linear and the temperature profile as quadratic (but not similar over the length). The pressure on the external boundary of the boundary layer is determined by the Newton method, allowing for the thickness of the displaced boundary layer. The surface temperature is assumed constant (absolutely heat-conducting body).

1. Formulation of Problem

Consider a flow past a slender cone at zero angle of attack. Assume: $M_\infty \gg 1$, $M_\infty \theta > 1$ (where θ is the vertex half-angle in radians), C_p = const, Pr = const, T_w = const, i.e., the external flow is hypersonic, the body absolutely heat-conducting, and the gas is perfect.

The equation of laminar boundary flow over the cone, taking account of the transverse curvature of the surface, has the form

$$\rho\left(u\frac{\partial u}{\partial \xi}+v\frac{\partial u}{\partial \eta}\right)=-\frac{\partial p}{\partial \xi}+\frac{\partial}{\partial \eta}\left(\mu\frac{\partial u}{\partial \eta}\right)+\frac{\mu}{r+\eta}\frac{\partial u}{\partial \eta},$$

$$\frac{\partial p}{\partial \eta}=0,\ i=JC_pT,\ p=R\rho T,$$

$$\rho\left(u\frac{\partial i}{\partial \xi}+v\frac{\partial i}{\partial \eta}\right)=u\frac{\partial p}{\partial \xi}+\frac{\partial}{\partial \eta}\left(\frac{\mu}{\mathrm{Pr}}\frac{\partial i}{\partial \eta}\right)+\frac{\mu}{\mathrm{Pr}\,(r+\eta)}\frac{\partial i}{\partial \eta}+\mu\left(\frac{\partial u}{\partial \eta}\right)^2,$$

$$\frac{\partial}{\partial \xi}[\rho u(r+\eta)]+\frac{\partial}{\partial \eta}[\rho v(r+\eta)]=0.$$

Here, ξ is the coordinate along the generatrix of the cone, η the coordinate along the normal to the surface, u, v the velocity components in the directions ξ, η, and r the local radius of the cone.

Let us determine the pressure on the body by the Newton method. If we take into account δ^*, which is the thickness of the displaced boundary layer, then the expression for the pressure will read

---

* Numbers given in the margin indicate pagination in the original foreign text.

$$p = \rho_\infty u^2_\infty \left(\theta + \frac{d\delta^*}{d\xi}\right)^2. \tag{1.1}$$

We note that, under the assumptions adopted, $u_\delta \approx u_\infty$, while $\delta^* \approx \delta$.

It is easy to obtain the integral relations of momentum and energy; after eliminating a number of terms - which the formulation of the problem permits - these relations take the form:

$$\frac{d}{d\xi}\int_0^\delta \rho u (r+\eta)(u_\delta - u)\, d\eta = \delta (r + \delta/2)\frac{\partial p}{d\xi} + r\left(\mu \frac{\partial u}{\partial \eta}\right)_w, \tag{1.2a}$$

/10

$$\frac{d}{d\xi}\int_0^\delta \rho u (r+\eta)(\tau_\delta - \tau)\, d\eta = \frac{r}{\Pr}\left(\mu \frac{\partial \tau}{\partial \eta}\right)_w. \tag{1.2b}$$

Here, $\tau = i + u^2/2$.

2. Approximation of the Velocity and Temperature Profiles

Let us take the profile of the longitudinal velocity component as linear, transverse to the boundary layer:

$$\frac{u}{u_\delta} = \frac{\eta}{\delta}. \tag{2.1}$$

Let us assume the temperature profile as quadratic:

$$\frac{i}{\tau_\infty} = f\left(1 - \frac{\eta}{\delta}\right)\left(1 + \frac{\eta}{t\delta}\right) \tag{2.2}$$

[an analogous approximation was used by Pai (Ref.1)].

In the resultant formula, t depends on ξ, i.e., there is no similarity of the temperature profile over the length, while f is the dimensionless temperature of the wall, $f = 2i_w/u_\infty^2$.

The approximation used for the velocity and temperature profiles is unsatisfactory in the case of a strongly cooled surface. For $f \ll 1$, u and i cannot be represented at all in the form of simple polynomials in η. Let us try, for example, to satisfy the equation of influx of heat to the wall:

$$\frac{\partial}{\partial \eta}\left(\frac{\mu}{\Pr}\frac{\partial i}{\partial \eta}\right) + \frac{\mu}{\Pr(r+\eta)}\frac{\partial i}{\partial \eta} + \mu\left(\frac{\partial u}{\partial \eta}\right)^2 = 0.$$

Let us represent u and i in the form of series:

$$u = u_\infty [a_1 \eta/\delta + a_2 (\eta/\delta)^2 + \ldots], \; i = \tau_\infty [f + b_1 \eta/\delta + b_2 (\eta/\delta)^2 + \ldots].$$

Substituting the series for u and i into the equation for the influx of heat to the wall, we obtain the following expression for f:

$$f = - \frac{0.5\, b_1^2}{2\,\mathrm{Pr}\, a_1^2 + 2 b_2 + b_1 \delta/r}.$$

An analysis of this formula shows that, at $f \ll 1$, it is impossible for a_1, b_1, and b_2 to be all of the order of unity at the same time.

Thus the approximations adopted for i and u are not satisfactory for $f \ll 1$, although the use of the integral relations will probably permit some extension of the range of value of f at which the results of this work can be used.

3. Solution of the System of Equations

Substituting eqs.(2.1), (2.2) and (1.1) into eq.(1.2), we obtain a system of two second-order differential equations. At $\xi = 0$ the equations have a singularity. Let us perform the substitution:

$$x = (\rho_\infty u_\infty \theta^4 \xi/\mu_w)^{1/5}, \; y = \delta (\rho_\infty u_\infty/\mu_w \theta \xi^4)^{1/5}.$$

As a result, we obtain a dimensionless system of differential equations not containing singularities at x = 0:

$$y\left\{x \frac{d}{dx}\left[\left(4y+5x+x\frac{dy}{dx}\right)^2 y(Ay+Bx)\right]+6\left(4y+5x+x\frac{dy}{dx}\right)^2 y(Ay+Bx)\right\}-$$
$$-y^2\left(x+\frac{y}{2}\right)\left\{x\frac{d}{dx}\left(4y+5x+x\frac{dy}{dx}\right)^2-\right.$$
$$\left.-2\left(4y+5x+x\frac{dy}{dx}\right)^2\right\}-125=0. \tag{3.1a}$$

$$y\left\{x\frac{d}{dx}\left[\left(4y+5x+x\frac{dy}{dx}\right)^2 y(Ly+Mx)\right]+\right.$$ /11
$$\left.+6\left(4y+5x+x\frac{dy}{dx}\right)^2 y(Ly+Mx)\right\}-125=0. \tag{3.1b}$$

In eq.(3.1), A, B, L and M are functions of t at constant Pr, f and $\varkappa$:

$$A = \frac{2\varkappa}{f(\varkappa-1)}\left[\frac{t}{2} - t^2 + t^3 \ln\left(1+\frac{1}{t}\right)\right],$$
$$B = \frac{2\varkappa}{f(\varkappa-1)}\left[t - t^2 \ln\left(1+\frac{1}{t}\right)\right],$$

$$L = \frac{2\varkappa \Pr}{f^2(\varkappa-1)}\left[-t^3 + \frac{t^2}{2} - \frac{t}{3} + \frac{t(1-f)}{3(1-t)} + t^4 \ln\left(1+\frac{1}{t}\right)\right],$$
$$M = \frac{2\varkappa \Pr}{f^2(\varkappa-1)}\left[t^2 - \frac{t}{2} + \frac{t(1-f)}{2(1-t)} - t^3 \ln\left(1+\frac{1}{t}\right)\right].$$

The system of equations (3.1) was solved numerically on an M-20 computer for several sets of values of f and Pr. For $0 < x < 0.2$, the system was solved by series, since the expressions for the derivatives, as $x \to 0$, are an indeterminacy of the form $O(x^2)/O(x^2)$. Let us represent y and t in the form of series:

$$y = y_0 + y_1 x + \ldots + y_m x^m + \ldots;$$
$$t = t_0 + t_1 x + \ldots + t_m x^m + \ldots$$

On substituting the series for y and t into eq.(3.1) and equating the coefficients of the various powers of x, we can successively determine all y_m and t_m (conventional method of indeterminate coefficients).

The first ten terms of the series were calculated. Taking account of such a large number of terms of the series is necessary to ensure a smooth transition to the numerical integration at $x > 0.2$. If the number of terms taken is small, the solution at $x > 0.2$ may prove unstable, due to the small errors produced in solving the series for $x < 0.2$.

The formulas for determining y_m and t_m for large m are extremely unwieldy and cannot be directly programmed. We did, however, succeed in setting up a general logical scheme for calculating y_m and t_m by the method of indeterminate coefficients, without separately programming the calculation of each individual coefficient.

In the range of $0.2 < x < 4$, the system (3.1) was solved numerically by the Adams method with the interval $\Delta x = 0.0005$. As a result we obtained the relation $y = y(x)$, which determines the law of thickness distribution of the boundary layer along the cone, and also $t = t(x)$.

4. Calculation of Pressure Distribution Coefficients of Frictional Drag and Heat Transfer

Knowing the relations $y = y(x)$ and $t = t(x)$, we can find the thickness of the displacement $\delta^* \approx \delta$, the pressure distribution over the surface of the cone, the coefficient of frictional drag C_F and the coefficient of heat transfer C_H.

We introduce the quantities C_F and C_H by means of the formulas

$$C_F = \frac{X}{q_\infty \pi r_l^2}, \quad C_H = \frac{Q_w}{\rho_\infty u_\infty \tau_\infty (1-f)\pi r_l^2}.$$

Here, X is the total frictional drag, Q_w the total heat flux to the cone surface, q_∞ the velocity head, r_l the maximum radius of the cone. In variables x, y, t the expressions for δ, p, C_F, C_H are of the form: /12

$$\delta = ry/x, \quad p = \rho_\infty u_\infty^2 \theta^2 (1 + 0.2\, dy/dx + 0.8\, y/x)^2,$$

$$C_F = \frac{20\,\theta^3}{\eta^{10}} \int_0^\eta \frac{x^6\, dx}{y}, \quad C_H = \frac{10\,\theta^2 f}{\eta^{10}\, \mathrm{Pr}\,(1-f)} \int_0^\eta \frac{(1-t)\, x^6\, dx}{ty}.$$

Here, η is the value of x at $\xi = l$.

For comparison, the quantities p, C_F and C_H were also calculated without allowing for the interaction of the boundary layer with the shock wave. All other assumptions adopted in the course of this work, however, were retained.

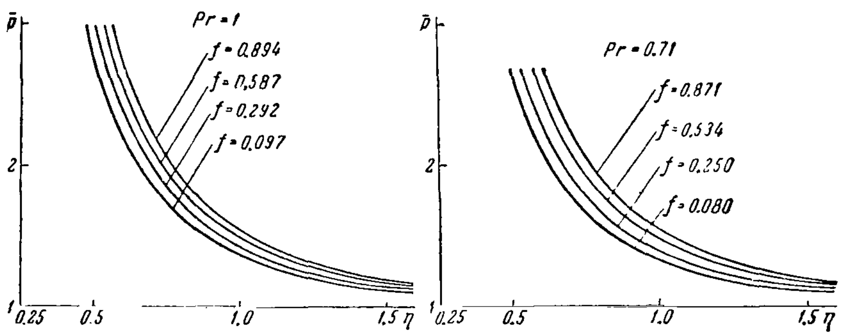

Fig.1

Figures 1 - 3 show the dependence of the relative quantities $\bar{p}$, $\bar{C}_F$, and $\bar{C}_H$ on the interaction parameter η. The quantities $\bar{p}$, $\bar{C}_F$, and $\bar{C}_H$ are the ratios of the values of p, C_F, and C_H calculated with allowance for the interaction of the boundary layer with the shock wave, to the corresponding values calculated without such allowance, and characterize the effect of the interaction.

5. Calculation of the Surface Temperature

The families of curves obtained at constant f permit, by the method of successive approximations, determining f itself, i.e., the surface temperature, if some method of heat removal from the surface of the body is assigned. It is not difficult to calculate the surface temperature if, for instance, the entire quantity of heat reaching the body from the gas by means of heat transfer is expended on radiation, proportional to the fourth power of the surface temperature. The equation of heat balance reads /13

$$\frac{C_H 8 (JC_p)^4 \rho_\infty \theta}{\varepsilon \sigma u_\infty^6} = \frac{f^4}{1-f}.$$

Here, σ is the Stefan-Boltzmann constant, and ϵ is the emissivity of the surface of the body. Two or three iterations have been shown by calculations to be sufficient for the process of successive approximation.

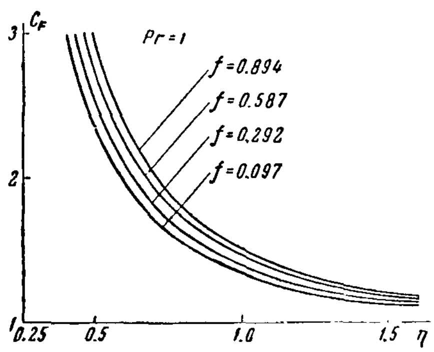

Fig.2

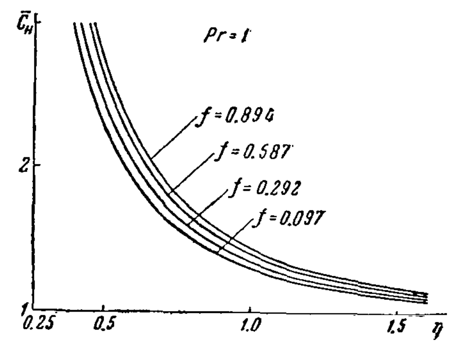

Fig.3

6. Limits of Applicability of the Method and a Few Conclusions

The above method permits taking account of the influence of interaction of the boundary layer over a slender cone with the shock wave in the entire range of interaction for cases of strong and weak interaction, and in the intermediate region. The fact that the method of calculation with and without allowance for the interaction is of the same kind permits the assumption that the relative quantities $\bar{p}$, $\bar{C}_F$, and $\bar{C}_H$ are found more accurately in this work than the absolute values of the corresponding quantities, whose determination is subject to the possible influence of a large number of simplifying assumptions.

Comparisons of the results of calculations without allowance for the interaction, with the more accurate calculations of C_F and C_H have shown discrepancies as high as 10 - 15% for C_F and 30 - 40% for C_H, in a number of cases (especially at $Pr \neq 1$).

The quantities C_F and C_H, however, may also be determined without allowing for the interaction; in that case, after calculating the interaction parameter η, these quantities can be multiplied by the relative values $\bar{C}_F$, $\bar{C}_H$, etc., obtained in this work.

It should also be borne in mind that the boundary-layer equations used in this work are applicable with an accurary of $(\delta/l)^2$, whereas the effect of slipping, in the case of $T_w = O(T_0)$ when it reaches a maximum is, according to estimates, of an order greater than $\lambda_w/\delta = O(\delta/l)$ (Ref.2), where λ_w is the mean free path of the molecules at the wall. This imposes certain restrictions in using the results of the present work.

The method presented may find application in calculations of the flow past slender cones (with a vertex half-angle of $\theta < 20^\circ$) at low Reynolds numbers in the range of flight altitudes of H = 50 - 100 km and in low-density wind tunnels.

REFERENCES

1. Pai, S.I. and Shen, S.F.: Hypersonic Viscous Flow over a Wedge. Proc. Midwestern Conf. Fluid Mech., 4th, 1955.
2. Galkin, V.S.: Study of the Hypersonic Flow of a Viscous, Slightly Rarefied Gas around a Flat Plate (Issledovaniye obtekaniya ploskoy plastiny giperzvukovym potokom vyazkogo slaborazrezhennogo gaza). Izv. Akad. Nauk SSSR, Otd. Tekhn. Nauk, Mekhanika i Mashinostr., No.3, 1961.

Translated for the National Aeronautics and Space Administration by the O.W.Leibiger Research Laboratories, Inc.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Notes, and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546